

2017—2018 学年第一学期《线性代数》A 卷参考答案

一、(30)

1. - 2. $\left(\frac{1}{2}, -1, 1\right)$ 3. $\frac{16}{3}$ 4. -4 5. $\begin{bmatrix} 1 & -3 \\ 3 & 2 \end{bmatrix}$ 6. 线性无关

7. 小于 n 8. $\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ 9. -8 10. $R(A) = R(A, b) < n$

二、(10)

$$D = \begin{vmatrix} 0 & -1 & -1 & 2 \\ 1 & 0 & 1 & 0 \\ -1 & 2 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{vmatrix} = 2 \times (-1)^{1+4} \begin{vmatrix} 1 & 0 & 1 \\ -1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} \text{-----}(5)$$

$$= -2 \begin{vmatrix} 1 & 0 & 0 \\ -1 & 2 & 0 \\ 2 & 1 & -1 \end{vmatrix} = -2 \times 1 \times (-1)^{1+1} \begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} = 4 \text{-----}(10)$$

三、(10)

$$[A, E] = \begin{bmatrix} 2 & 2 & 3 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 2 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 3 & 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 1 & 0 & 1 & 0 \\ 0 & -2 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix} \text{-----}(5)$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 & 1 & -1 & 0 \\ 0 & -2 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & -2 \\ 0 & -2 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 3 & -2 \\ 0 & 1 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & -2 & 1 \end{bmatrix} \text{-----}(9)$$

$$\therefore A^{-1} = \begin{bmatrix} 1 & 3 & -2 \\ -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & -2 & 1 \end{bmatrix} \text{-----}(10)$$

四、 (10)

$$X = A^{-1}B \text{-----}(2)$$

$$[A, B] = \begin{bmatrix} 1 & -1 & 1 & 1 & 1 & 2 \\ 2 & 1 & -1 & 2 & 1 & 1 \\ -1 & 0 & 1 & 1 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 1 & 1 & 1 & 2 \\ 0 & 3 & -3 & 0 & -1 & -3 \\ 0 & -1 & 2 & 2 & 3 & 3 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & \frac{2}{3} & 1 \\ 0 & 1 & -1 & 0 & -\frac{1}{3} & -1 \\ 0 & 0 & 1 & 2 & \frac{8}{3} & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & \frac{2}{3} & 1 \\ 0 & 1 & 0 & 2 & \frac{7}{3} & 1 \\ 0 & 0 & 1 & 2 & \frac{8}{3} & 2 \end{bmatrix} \text{-----}(8),$$

$$\therefore X = \begin{bmatrix} 1 & \frac{2}{3} & 1 \\ 2 & \frac{7}{3} & 1 \\ 2 & \frac{8}{3} & 2 \end{bmatrix} \text{-----}(10)$$

五、 (10)

$$\because \text{向量组线性相关}, \therefore \begin{vmatrix} 1 & 2 & 4 \\ 2 & a & 1 \\ -1 & 1 & -1 \end{vmatrix} = 0 \text{-----}(3)$$

$$\text{即} \begin{vmatrix} 1 & 2 & 4 \\ 2 & a & 1 \\ -1 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 4 \\ 0 & a-4 & -7 \\ 0 & 3 & 3 \end{vmatrix} = 0, \therefore 3(a-4) + 21 = 0, \therefore a = -3 \text{-----}(10)$$

六、 (15)

$$\begin{aligned}
 A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4) &= \begin{bmatrix} 2 & 1 & 2 & 3 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 2 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & 2 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & 2 \\ 4 & 1 & 3 & 5 \\ 2 & 0 & 1 & 2 \\ 2 & 1 & 2 & 3 \end{bmatrix} \\
 &\rightarrow \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -3 & -5 & -3 \\ 0 & -2 & -3 & -2 \\ 0 & -1 & -2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{-----}(7)
 \end{aligned}$$

$\therefore \alpha_1, \alpha_2, \alpha_3$ 为一个极大线性无关组, 且 $R(\alpha_1, \alpha_2, \alpha_3, \alpha_4) = 3$.-----(10)

$\therefore \alpha_4 = \alpha_1 + \alpha_2$ -----(15)

七、 (15)

$$[A, b] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 3 & 1 & 2 & 1 & 3 \\ 1 & 5 & 3 & 5 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & -2 & -1 & -2 & 0 \\ 0 & 4 & 2 & 4 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & \frac{1}{2} & 0 & 1 \\ 0 & 1 & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{-----}(7)$$

$$\text{得} \begin{cases} x_1 = -\frac{1}{2}x_3 + 1 \\ x_2 = -\frac{1}{2}x_3 - x_4 \end{cases}$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + k_1 \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, k_1, k_2 \text{ 为任意实数} \text{----- (15)}$$