第4章 不定积分习题参考答案

(经济数学)广东海洋大学，信计系

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3. 用换元法求下列不定积分.

(1) $\int \frac{dx}{3-2x}$;

解：原式=$-\frac{1}{2}\int \frac{d(3-2x)}{3-2x} = -\frac{1}{2}\ln|3-2x| + C$.

(2) $\int (2x-3)^{100}dx$;

解：原式=$\frac{1}{2}\int (2x-3)^{100}d(2x-3) = \frac{1}{202}(2x-3)^{101} + C$.

(3) $\int \frac{1}{x^2}e^{\frac{1}{x}}dx$;

解：原式=$-\int (\frac{1}{x})'e^{\frac{1}{x}}dx = -\int e^{\frac{1}{x}}d(\frac{1}{x}) = -e^{\frac{1}{x}} + C$.

(4) $\int \frac{cos\sqrt{x}}{\sqrt{x}}dx$;

解：原式=$2\int \cos\sqrt{x}(\sqrt{x})'dx = 2\int \cos\sqrt{x}d(\sqrt{x}) = 2\sin\sqrt{x} + C$.

(5) $\int \frac{dx}{x\ln x}$;

解：原式=$\int \frac{1}{\ln x}\frac{1}{x}dx = \int \frac{1}{\ln x}d(\ln x) = \ln|\ln x| + C$.

(6) $\int \frac{\cos x}{\sin^2 x}$;

解：原式=$\int \frac{1}{\sin^2 x}d(\sin x) = -\frac{1}{\sin x} + C$.

(7) $\int x\cos(x^2)dx$;

解：原式=$\frac{1}{2}\int \cos(x^2)d(x^2) = \frac{1}{2}\sin(x^2) + C$.

(8) $\int x\sin(x^2)dx$; 注：教材答案有误

解：原式=$\frac{1}{2}\int \sin(x^2)d(x^2) = -\frac{1}{2}\cos(x^2) + C$.

(9)$\int e^x \sin(e^x)$;

解：原式=$\int \sin(e^x)d(e^x) = -\cos(e^x) + C$.

(10)$\int \frac{dx}{\sqrt{x}(1+x)}$;

解：原式=$\int \frac{1}{1+(\sqrt{x})^2}\frac{1}{\sqrt{x}}dx = 2\int \frac{1}{1+(\sqrt{x})^2}d(\sqrt{x}) = 2\arctan\sqrt{x} + C$.

(11)$\int x^2\sqrt{1+x^3}dx$;

解：原式=$\frac{1}{3}\int (1+x^3)'\sqrt{1+x^3}dx = \frac{1}{3}\int \sqrt{1+x^3}d(1+x^3) = \frac{2}{9}(1+x^3)^{\frac{3}{2}} + C$.

(12)$\int \frac{dx}{\sin x\cos x}$;

解：分子分母同除以$\cos^2 x$. 原式=$\int \frac{\sec^2 x}{\tan x}dx = \int \frac{1}{\tan x}d(\tan x) = \ln|\tan x| + C$.

(13)$\int \tan\sqrt{1+x^2}\frac{x}{\sqrt{1+x^2}}dx$;

解：原式=$\int \tan\sqrt{1+x^2}(\sqrt{1+x^2})'dx = \int \tan\sqrt{1+x^2}d(\sqrt{1+x^2})$

$= -\ln|\cos\sqrt{1+x^2}| + C$.

(14)$\int \frac{\sin x\cos x}{1+\sin^4 x}dx$;

解：原式=$\frac{1}{2}\int \frac{(\sin^2 x)'}{1+(\sin^2 x)^2}dx = \frac{1}{2}\int \frac{d(\sin^2 x)}{1+(\sin^2 x)^2} = \frac{1}{2}\arctan(\sin^2 x) + C$.

(15)$\int \frac{dx}{x\sqrt{x^2-1}}$;

法一：原式=$\int \frac{1}{x^2\sqrt{1-\frac{1}{x^2}}} = -\int \frac{1}{\sqrt{1-\frac{1}{x^2}}}(\frac{1}{x})'dx = -\int \frac{1}{\sqrt{1-\frac{1}{x^2}}}d(\frac{1}{x})$

$= -\arcsin\frac{1}{x} + C = \arccos\frac{1}{x} + C$;

法二：令$x = \sec t, dx = \sec t\tan t dt$，则

原式=$\int \frac{1}{\sec t\tan t}\sec t\tan t dt = t + C = \arccos\frac{1}{x} + C$.

法三：令$\sqrt{x^2-1} = t$, 则$x^2 - 1 = t^2, 2xdx = 2tdt$,

原式=$\int \frac{2xdx}{2x^2\sqrt{x^2-1}} = \int \frac{2tdt}{2(t^2+1)t} = \int \frac{1}{1+t^2}dt = \arctan t + C$

$=\arctan\sqrt{x^2-1} + C = \arccos\frac{1}{x} + C$.

法四：令$\frac{1}{x} = t, x = \frac{1}{t}, dx = -\frac{1}{t^2}dt$. 则

原式$=\int \frac{1}{\frac{1}{t}\sqrt{\frac{1}{t^2}-1}}(-\frac{1}{t^2})dt = -\int \frac{dt}{\sqrt{1-t^2}} = -\arcsin t + C = -\arcsin\frac{1}{x} + C.$

(16)$\int \frac{dx}{2x^2-1}$;

解：原式$=\int \frac{dx}{(\sqrt{2}x+1)(\sqrt{2}x-1)} = \frac{1}{2}\int \frac{(\sqrt{2}x+1)-(\sqrt{2}x-1)}{(\sqrt{2}x+1)(\sqrt{2}x-1)}dx$

$$=\frac{1}{2}\int(\frac{1}{\sqrt{2}x-1}-\frac{1}{\sqrt{2}x+1})dx = \frac{1}{2}\int\frac{dx}{\sqrt{2}x-1} - \frac{1}{2}\int\frac{dx}{\sqrt{2}x+1}$$

$$=\frac{1}{2\sqrt{2}}\int\frac{d(\sqrt{2}x-1)}{\sqrt{2}x-1} - \frac{1}{2\sqrt{2}}\int\frac{d(\sqrt{2}x+1)}{\sqrt{2}x+1} = \frac{1}{2\sqrt{2}}\ln|\frac{\sqrt{2}x-1}{\sqrt{2}x+1}| + C.$$

(17)$\int \tan^4 x dx$;

解：原式$=\int \tan^2 x\tan^2 x dx = \int \tan^2 x(\sec^2 x - 1)dx = \int \tan^2 x\sec^2 x dx - \int \tan^2 x dx$

$$=\int \tan^2 x d(\tan x) - \int(\sec^2 x - 1)dx = \frac{1}{3}\tan^3 x - \tan x + x + C.$$

(18)$\int \cos^5 x dx$;

解：原式$=\int \cos^4 x\cos x dx = \int (1-\sin^2 x)^2\cos x dx = \int(1-2\sin^2 x+\sin^4 x)d(\sin x)$

$$=\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + C.$$

(19)$\int \frac{2x-1}{\sqrt{1-x^2}}dx$;

解：令$x=\sin t, t\in(-\frac{\pi}{2},\frac{\pi}{2})$,则$\sqrt{1-x^2}=\cos t, dx=\cos t dt.$

原式$=\int \frac{2\sin t-1}{\cos t}\cos t dt = \int(2\sin t-1)dt = -2\cos t - t + C = -2\sqrt{1-x^2} - \arcsin x + C.$

(20)$\int \frac{1-x}{\sqrt{9-4x^2}}dx$;

解：原式$=\int \frac{1}{\sqrt{9-4x^2}}dx - \int\frac{x}{\sqrt{9-4x^2}}dx = \int\frac{1}{2\sqrt{(\frac{3}{2})^2-x^2}}dx + \frac{1}{8}\int\frac{d(9-4x^2)}{\sqrt{9-4x^2}}$

$$=\frac{1}{2}\arcsin\frac{2x}{3} + \frac{1}{4}\sqrt{9-4x^2} + C.$$

(21)$\int \tan^3 x\sec x dx$;

解：原式$=\int \tan^2 x\tan x\sec x dx = \int(\sec^2 x - 1)d(\sec x) = \frac{1}{3}\sec^3 x - \sec x + C.$

(22)$\int 10^{\arcsin x}\frac{dx}{\sqrt{1-x^2}}$;

解：原式$=\int 10^{\arcsin x}(\arcsin x)'dx = \int 10^{\arcsin x}d(\arcsin x) = \frac{1}{\ln 10}10^{\arcsin x} + C.$

(23) $\int \frac{\sin x+\cos x}{\sqrt[3]{\sin x-\cos x}}dx;$

解：原式$=\int \frac{(\sin x-\cos x)'}{\sqrt[3]{\sin x-\cos x}}dx=\int \frac{1}{\sqrt[3]{\sin x-\cos x}}d(\sin x-\cos x)=\frac{3}{2}(\sin x-\cos x)^{\frac{2}{3}}+C.$

(24) $\int \frac{1+\ln x}{(x\ln x)^2}dx;$

解：原式$=\int \frac{(x\ln x)'}{(x\ln x)^2}dx=\int \frac{1}{(x\ln x)^2}d(x\ln x)=-\frac{1}{x\ln x}+C.$

(25) $\int \frac{dx}{x^2+x+1};$

解：原式$=\int \frac{1}{(x+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}d(x+\frac{1}{2})=\frac{2}{\sqrt{3}}\arctan\frac{2x+1}{\sqrt{3}}+C.$

(26) $\int \frac{dx}{x^2+2x+2};$

解：原式$=\int \frac{d(x+1)}{(x+1)^2+1}=\arctan(x+1)+C.$

(27) $\int \frac{dx}{(x+1)(x-2)};$

解：原式$=\frac{1}{3}\int \frac{(x+1)-(x-2)}{(x+1)(x-2)}dx=\frac{1}{3}\int (\frac{1}{x-2}-\frac{1}{x+1})dx$

$=\frac{1}{3}(\ln|x-2|-\ln|x+1|)+C=\frac{1}{3}\ln|\frac{x-2}{x+1}|+C.$

(28) $\int \frac{dx}{x(1+x)};$

解：原式$=\int \frac{(x+1)-x}{x(1+x)}dx=\int (\frac{1}{x}-\frac{1}{1+x})dx=\ln|x|-\ln|1+x|+C=\ln|\frac{x}{1+x}|+C.$

(29) $\int \cos x\cos 2x dx;$

解：原式$=\int \cos x(1-2\sin^2 x)dx=\int (1-2\sin^2 x)d(\sin x)=\sin x-\frac{2}{3}\sin^3 x+C.$

另法（积化和差）：原式$=\frac{1}{2}\int (\cos 3x+\cos x)dx=\frac{1}{6}\sin 3x+\frac{1}{2}\sin x+C.$ $(\sin 3x=3\sin x-4\sin^3 x).$

(30) $\int \sin 5x\sin 7x dx;$ 注：教材答案有误

解：原式$=-\frac{1}{2}\int (\cos 12x-\cos 2x)dx=-\frac{1}{24}\sin 12x+\frac{1}{4}\sin 2x+C.$

(31) $\int \frac{x^2}{\sqrt{1-x^2}}dx;$

解：令$x=\sin t,\ t\in(-\frac{\pi}{2},\frac{\pi}{2})$, 则$dx=\cos t dt.$

原式$=\int \frac{\sin^2 t}{\cos t}\cos t dt=\int \sin^2 t dt=\int \frac{1-\cos 2t}{2}dt=\frac{t}{2}-\frac{1}{4}\sin 2t+C=\frac{t}{2}-\frac{1}{2}\sin t\cos t+C$

$$=\frac{1}{2}\arcsin x-\frac{x}{2}\sqrt{1-x^2}+C.$$

(32) $\int\frac{dx}{\sqrt{a^2-x^2}}(a>0)$; 注：教材答案有误

解：令$x=a\sin t, t\in(-\frac{\pi}{2},\frac{\pi}{2})$，则$dx=a\cos tdt$, $\sqrt{a^2-x^2}=a\cos t$.

原式$=\int\frac{1}{a\cos t}a\cos tdt=\int 1dt=t+C=\arcsin\frac{x}{a}+C.$

(33) $\int\frac{dx}{x^2\sqrt{1+x^2}}$;

解：令$x=\tan t, dx=\sec^2 tdt, \sqrt{1+x^2}=\sec t.$

原式$=\int\frac{1}{\tan^2\sec t}\sec^2 tdt=\int\frac{\sec t}{\tan^2 t}dt=\int\frac{\cos t}{\sin^2 t}dt=\int\frac{d(\sin t)}{\sin^2 t}=-\frac{1}{\sin t}+C=-\frac{\sqrt{1+x^2}}{x}+C.$

(34) $\int\frac{dx}{\sqrt{(x^2+1)^3}}$;

解：令$x=\tan t, x^2+1=\sec^2 t, dx=\sec^2 tdt,$

原式$=\int\frac{\sec^2 t}{\sec^3 t}dt=\int\cos tdt=\sin t+C=\frac{x}{\sqrt{1+x^2}}+C.$

(35) $\int\frac{\sqrt{x^2-9}}{x}dx$;

解：令$x=3\sec t, \sqrt{x^2-9}=3\tan t, dx=3\sec t\tan tdt,$

原式$=\int\frac{3\tan t}{3\sec t}3\sec t\tan tdt=3\int\tan^2 tdt=3\int(\sec^2 t-1)dt=3(\tan t-t)+C$

$$=\sqrt{x^2-9}-3\arccos\frac{3}{x}+C.$$

(36) $\int\frac{dx}{\sqrt{(x^2-a^2)^3}}(a>0)$;

解：令$x=a\sec t, dx=a\sec t\tan tdt,$

原式$=\int\frac{a\sec t\tan t}{(a\tan t)^3}dt=\int\frac{\sec t}{a^2\tan^2 t}dt=\frac{1}{a^2}\int\frac{\cos t}{\sin^2 t}dt=-\frac{1}{a^2}\frac{1}{\sin t}+C=-\frac{1}{a^2}\frac{x}{\sqrt{x^2-a^2}}+C.$

(37) $\int\frac{dx}{1+\sqrt{2x}}$;

解：令$\sqrt{2x}=t, 2x=t^2, x=\frac{t^2}{2}, dx=tdt,$

原式$=\int\frac{t}{1+t}dt=\int(1-\frac{1}{1+t})dt=t-\ln|1+t|+C=\sqrt{2x}-\ln(1+\sqrt{2x})+C.$

(38) $\int\frac{\sqrt{x}}{\sqrt[3]{x}-\sqrt{x}}dx$;

解：令$\sqrt[6]{x}=t, x=t^6, dx=6t^5dt,$

原式$=\int\frac{t^3}{t^4-t^3}6t^5dt=\int\frac{6t^5}{t-1}dt=6\int(\frac{t^5-1}{t-1}+\frac{1}{t-1})dt=6\int(t^4+t^3+t^2+1+\frac{1}{t-1})dt$

$$=6(\frac{t^5}{5}+\frac{t^4}{4}+\frac{t^3}{3}+t+\ln|t-1|)+C=6(\frac{\sqrt[6]{x^5}}{5}+\frac{\sqrt[3]{x^2}}{4}+\frac{\sqrt{x}}{3}+\sqrt[6]{x}+\ln|\sqrt[6]{x}-1|)+C.$$

(39) $\int \frac{dx}{(x+2)\sqrt{x+1}}$;

解：令 $\sqrt{x+1}=t, x+1=t^2, x=t^2-1, dx=2tdt$,

原式 $=\int \frac{2tdt}{(t^2+1)t}=2\int \frac{dt}{1+t^2}=2\arctan t+C=2\arctan\sqrt{x+1}+C$.

(40) $\int \frac{dx}{\sqrt{1+e^x}}$;

解：令 $\sqrt{1+e^x}=t, e^x=t^2-1, x=\ln(t^2-1), dx=\frac{2t}{t^2-1}dt$,

原式 $=\int \frac{1}{t}\frac{2t}{t^2-1}dt=2\int \frac{1}{t^2-1}dt=\int \frac{(t+1)-(t-1)}{(t+1)(t-1)}dt=\int (\frac{1}{t-1}-\frac{1}{t+1})dt$

$=\ln|\frac{t-1}{t+1}|+C=\ln\frac{\sqrt{1+e^x}-1}{\sqrt{1+e^x}+1}+C$.

4. 用分部积分法求下列不定积分.

(1) $\int x\sin 2xdx$;

解：原式 $=\frac{1}{2}x\cos 2x+\frac{1}{2}\int \cos 2xdx=-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x+C$.

(2) $\int xe^{-x}dx$;

解：原式 $=-xe^{-x}+\int e^{-x}dx=-xe^{-x}-e^{-x}+C$.

(3) $\int \ln xdx$;

解：原式 $=x\ln x-\int x\frac{1}{x}dx=x\ln x-x+C$.

(4) $\int x^2\cos 3xdx$;

解：原式 $=\frac{1}{3}x^2\sin 3x-\int \frac{2}{3}x\sin 3xdx=\frac{1}{3}x^2\sin 3x-(-\frac{2}{9}x\cos 3x+\int \frac{2}{9}\cos 3xdx)$

$=\frac{1}{3}x^2\sin 3x+\frac{2}{9}x\cos 3x-\frac{2}{27}\sin 3x+C$.

(5) $\int e^{-x}\sin xdx$;

解：原式 $=-e^{-x}\cos x+\int (-e^{-x})\cos xdx=-e^{-x}\cos x-(e^{-x}\sin x+\int e^{-x}\sin xdx)$

$\therefore$ 原式 $=-\frac{1}{2}e^{-x}(\cos x+\sin x)+C$.

(6) $\int x^2\arctan xdx$;

解：原式 $=\frac{x^3}{3}\arctan x-\int \frac{x^3}{3}\frac{1}{1+x^2}dx=\frac{x^3}{3}\arctan x-\frac{1}{3}\int \frac{x^3+x-x}{1+x^2}dx$

$$=\frac{x^3}{3}\arctan x-\frac{1}{3}\int(x-\frac{x}{1+x^2})dx=\frac{x^3}{3}\arctan x-\frac{x^2}{6}+\frac{1}{6}\ln(1+x^2)+C.$$

(7) $\int\frac{(\ln x)^3}{x^2}dx$;

解：原式$=-\frac{1}{x}(\ln x)^3+\int\frac{1}{x}3(\ln x)^2\frac{1}{x}dx=-\frac{1}{x}(\ln x)^3+3\int\frac{1}{x^2}(\ln x)^2dx$

$$=-\frac{1}{x}(\ln x)^3+3[-\frac{1}{x}(\ln x)^2+\int\frac{2\ln x}{x^2}dx]=-\frac{1}{x}(\ln x)^3-\frac{3}{x}(\ln x)^2-6\int\ln x(\frac{1}{x})'dx$$

$$=-\frac{1}{x}(\ln x)^3-\frac{3}{x}(\ln x)^2-6(\frac{1}{x}\ln x-\int\frac{1}{x}\frac{1}{x}dx)=-\frac{1}{x}[(\ln x)^3+3(\ln x)^2+6\ln x+6]+C.$$

(8) $\int(\arcsin x)^2dx$;

解：原式$=x(\arcsin x)^2-2\int x\arcsin x\frac{1}{\sqrt{1-x^2}}dx=x(\arcsin x)^2-2\int\arcsin x\frac{x}{\sqrt{1-x^2}}dx$

$$=x(\arcsin x)^2+2\sqrt{1-x^2}\arcsin x-2\int\frac{1}{\sqrt{1-x^2}}\sqrt{1-x^2}dx$$

$$=x(\arcsin x)^2+2\sqrt{1-x^2}\arcsin x-2x+C.$$

另法：令$x=\sin t$, 则$\arcsin x=t, dx=costdt$，代入原式再用分部积分法。

(9)$\int x\tan^2 xdx$;

解：原式$=\int x(\sec^2 x-1)dx=\int x\sec^2 xdx-\int xdx=x\tan x-\int\tan xdx-\frac{x^2}{2}+C$

$$=x\tan x+\ln|\cos x|-\frac{x^2}{2}+C$$

(10) $\int\cos(\ln x)dx$;

解：令$\ln x=t, x=e^t, dx=e^tdt$,

原式$=\int e^t\cos tdt=e^t\ sint-\int e^t\sin tdt=e^t\sin t-(-e^t\cos t+\int e^t\cos tdt)$

$$\therefore\ \int e^t\cos tdt=\frac{1}{2}e^t(\sin t+\cos t)+C=\frac{x}{2}[\sin(\ln x)+\cos(\ln x)]+C.$$

(11) $\int\frac{\arcsin\sqrt{x}}{\sqrt{1-x}}dx$;

解：原式$=-2\sqrt{1-x}\arcsin\sqrt{x}+\int 2\sqrt{1-x}\frac{1}{\sqrt{1-x}}\frac{1}{2\sqrt{x}}dx=-2\sqrt{1-x}\arcsin\sqrt{x}+\int\frac{1}{\sqrt{x}}dx$

$$=-2\sqrt{1-x}\arcsin\sqrt{x}+2\sqrt{x}+C.$$

另法：令$x=\sin^2 t$, 则$\arcsin\sqrt{x}=t,\sqrt{1-x}=\cos t, dx=2\sin t\cos tdt$,

原式$=\int\frac{t}{\cos t}2\sin t\cos tdt=\int 2t\sin tdt=-2t\cos t+2\int\cos tdt=-2t\cos t+2\sin t+C$

$$=-2\sqrt{1-x}\arcsin\sqrt{x}+2\sqrt{x}+C.$$

(12) $\int\ln(x+\sqrt{1+x^2})dx$;

解：原式$=x\ln(x+\sqrt{1+x^2})-\int x\frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}dx=x\ln(x+\sqrt{1+x^2})-\int\frac{x}{\sqrt{1+x^2}}dx$

$=x\ln(x+\sqrt{1+x^2})-\frac{1}{2}\int\frac{d(1+x^2)}{\sqrt{1+x^2}}=x\ln(x+\sqrt{1+x^2})-\sqrt{1+x^2}+C.$

5.求下列有理函数的积分.

(1) $\int\frac{dx}{x(x^2+1)}$;

解：原式$=\int\frac{(x^2+1)-x^2}{x(x^2+1)}dx=\int(\frac{1}{x}-\frac{x}{x^2+1})dx=\ln|x|-\frac{1}{2}\ln|x^2+1|+C.$

$=\ln\frac{x}{\sqrt{1+x^2}}+C.$

(2) $\int\frac{dx}{(1+x)(1+x^2)}$;

解：$\frac{1}{(1+x)(1+x^2)}=\frac{a}{1+x}+\frac{bx+c}{1+x^2}=\frac{a(1+x^2)+(1+x)(bx+c)}{(1+x)(1+x^2)}$

$=\frac{(a+b)x^2+(b+c)x+(a+c)}{(1+x)(1+x^2)}.$

$\therefore\ a+b=0,b+c=0,a+c=1;\therefore\ b=-\frac{1}{2},a=\frac{1}{2},c=\frac{1}{2};$

$\therefore$ 原式$=\frac{1}{2}\int(\frac{1}{1+x}+\frac{-x+1}{1+x^2})dx=\frac{1}{2}\ln|1+x|+\frac{1}{2}\int\frac{-x}{1+x^2}dx+\frac{1}{2}\int\frac{dx}{1+x^2}$

$=\frac{1}{2}\ln|1+x|-\frac{1}{4}\ln(1+x^2)+\frac{1}{2}\arctan x+C.$ 注：教材答案有误。

(3) $\int\frac{2x-1}{(x-1)(x-2)}dx$;

解：原式$=\int\frac{3(x-1)-(x-2)}{(x-1)(x-2)}dx=\int(\frac{3}{x-2}-\frac{1}{x-1})dx=3\ln|x-2|-\ln|x-1|+C.$

(4) $\int\frac{2x+3}{x^2+3x-10}dx$;

解：原式$=\int\frac{2x+3}{(x+5)(x-2)}dx=\int\frac{(x+5)+(x-2)}{(x+5)(x-2)}dx=\int(\frac{1}{x+5}+\frac{1}{x-2})dx$

$=\ln|x+5|+\ln|x-2|+C.$

(5) $\int\frac{x^5+x^4-8}{x^3-x}dx$;

解：原式$=\int(x^2+x+1+\frac{8}{x}-\frac{4}{x+1}-\frac{3}{x-1})dx=\frac{x^3}{3}+\frac{x^2}{2}+x+8\ln|x|-4\ln|x+1|-3\ln|x-1|+C.$

(6) $\int\frac{x^2+1}{(x+1)^2(x^2+x)}dx$;

解：$\frac{x^2+1}{(x+1)^2(x^2+x)}=\frac{x^2+1}{x(x+1)^3}=\frac{a}{(x+1)^3}+\frac{b}{(x+1)^2}+\frac{c}{x+1}+\frac{d}{x}$

$=\frac{(a+b+c+3d)x+(b+2c+3d)x^2+(c+c)x^3+d}{x(x+1)^3}$

$\therefore a+b+c+3d=0, d=1, c+d=0, b+2c+3d=1, \therefore\ a=-2, b=0, c=-1, d=1;$

原式$=\int(\frac{-2}{(x+1)^3}-\frac{1}{x+1}+\frac{1}{x})dx=(x+1)^{-2}-\ln|x+1|+\ln|x|+C.$

注：教材答案有误。教材答案对应不定积分应为$\int\frac{dx}{(1+x^2)(x^2+x)}$.

6. 求下列三角函数的积分

(1) $\int\frac{dx}{3+\cos x}dx;$

解：令$u=\tan\frac{x}{2}$, 则$\cos x=\frac{1-u^2}{1+u^2}, dx=\frac{2}{1+u^2}du;$

原式$=\int\frac{1}{3+\frac{1-u^2}{1+u^2}}\frac{2}{1+u^2}du=\int\frac{2du}{3(1+u^2)+1-u^2}=\int\frac{2du}{4+2u^2}=\int\frac{du}{2+u^2}$

$=\int\frac{du}{(\sqrt{2})^2+u^2}=\frac{1}{\sqrt{2}}\arctan\frac{\tan\frac{x}{2}}{\sqrt{2}}+C.$

(2) $\int\frac{dx}{2+\sin x};$

解：令$u=\tan\frac{x}{2}$, 则$\sin x=\frac{2u}{1+u^2}, dx=\frac{2}{1+u^2}du;$

原式$=\int\frac{1}{2+\frac{2u}{1+u^2}}\frac{2}{1+u^2}du=\int\frac{1}{u^2+u+1}du=\int\frac{d(u+\frac{1}{2})}{(u+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$

$=\frac{2}{\sqrt{3}}\arctan\frac{2u+1}{\sqrt{3}}+C=\frac{2}{\sqrt{3}}\arctan\frac{2\tan\frac{x}{2}+1}{\sqrt{3}}+C.$

(3) $\int\frac{dx}{1+\sin x+\cos x};$

解：原式$=\int\frac{dx}{1+\cos x+\sin x}=\int\frac{dx}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}=\int\frac{\sec^2\frac{x}{2}}{2+2\tan\frac{x}{2}}dx$

$=\int\frac{(1+\tan\frac{x}{2})'}{1+\tan\frac{x}{2}}dx=\int\frac{1}{1+\tan\frac{x}{2}}d(1+\tan\frac{x}{2})=\ln|1+\tan\frac{x}{2}|+C.$

注：也可用换元法，令$u=\tan\frac{x}{2}$.

(4) $\int\sin^4 x\cos^5 x dx;$

解：原式$=\int\sin^4 x\cos^4 x\cos x dx=\int\sin^4 x(1-\sin^2 x)^2 d(\sin x)=\int(\sin^4 x-2\sin^6 x+\sin^8 x)d(\sin x)$

$=\frac{1}{5}\sin^5 x-\frac{2}{7}\sin^2 x+\frac{1}{9}\sin^9 x+C.$

(5) $\int\frac{\sin x}{1+\sin x}dx;$

解：原式$=\int\frac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}dx=\int\frac{\sin x-\sin^2 x}{\cos^2 x}dx=\int(\tan x\sec x-\tan^2 x)dx$

$=\sec x-\int(\sec^x-1)dx=\sec x-\tan x+x+C.$

(6) $\int \frac{\sin^2 x}{\cos^3 x}dx$;

解：原式$=\int \frac{\sin^2 x}{\cos^2 x}\frac{1}{\cos x}dx = \int \tan^2 x \sec x dx = \int (\tan x \sec x)\tan x dx$;

$=\sec x \tan x - \int \sec x \sec^2 x dx = \sec x \tan x - \int \sec x(\tan^2 x + 1)dx$

$=\sec x \tan x - \int \sec x \tan^2 x dx - \int \sec x dx$

$\therefore$ 原式$=\int \sec x \tan^2 x dx = \frac{1}{2}(\sec x \tan x - \int \sec x dx) = \frac{1}{2}(\sec x \tan x - \ln|\sec x + \tan x|) + C.$

7. 求下列简单无理函数的积分.

(1) $\int \frac{x^3}{1+\sqrt{1+x^4}}dx$;

解：令$\sqrt{1+x^4} = t, x^4 = t^2 - 1, 4x^3 dx = 2tdt$;

$\therefore$ 原式$=\int \frac{\frac{t}{2}}{1+t}dt = \frac{1}{2}\int \frac{1+t-1}{1+t}dt = \frac{1}{2}\int (1 - \frac{1}{1+t})dt$

$=\frac{1}{2}(t - \ln|1+t|) + C = \frac{1}{2}(\sqrt{1+x^4} - \ln(1+\sqrt{1+x^4}) + C.$

(2) $\int \frac{dx}{1+\sqrt[3]{x+1}}$;

解：令$\sqrt[3]{x+1} = t, x = t^3 - 1, dx = 3t^2 dt$;

原式$=3\int \frac{t^2}{1+t}dt = 3\int \frac{t^2-1+1}{1+t}dt = 3\int [(t-1) + \frac{1}{1+t}]dt = 3(\frac{t^2}{2} - t + \ln|1+t|) + C$

$=3(\frac{\sqrt[3]{(x+1)^2}}{2} - \sqrt[3]{x+1} + \ln|1+\sqrt[3]{x+1}|) + C.$

(3) $\int \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}dx$;

解：令$\sqrt{x+1} = t, x = t^2 - 1, dx = 2tdt$;

原式$=\int \frac{t-1}{t+1}2tdt = \int \frac{t+1-2}{t+1}2tdt = \int 2t(1 - \frac{2}{t+1})dt = \int (2t - \frac{4t}{t+1})dt$

$=\int (2t - \frac{4(t+1)-4}{t+1})dt = \int (2t - 4 + \frac{4}{t+1})dt = t^2 - 4t + 4\ln|t+1| + C$

$=x - 4\sqrt{x+1} + 4\ln(1+\sqrt{x+1}) + C$. (注：$t^2 = x+1$，这里的1被合并到任意常数$C$中。)

(4) $\int \frac{x\sqrt{x}}{\sqrt{x}+1}dx$;

解：令$\sqrt{x} = t, x = t^2, dx = 2tdt$;

原式$=\int \frac{t^3+1}{t+1}2tdt = \int (t^2 - t + 1)2tdt = \int (2t^3 - 2t^2 + 2t)dt$

$=\frac{1}{2}t^4 - \frac{2}{3}t^3 + t^2 + C = \frac{1}{2}x^2 - \frac{2}{3}x^{\frac{3}{2}} + x + C.$

另法：原式=$\int \frac{x\sqrt{x}+x-x+1}{\sqrt{x}+1}dx = \int (x-\frac{x-1}{\sqrt{x}+1})dx = \int (x-(\sqrt{x}-1))dx = \frac{x^2}{2}-\frac{2}{3}x^{\frac{3}{2}}+x+C.$

8. 设某厂生产某种机床x台的总成本C是产量的函数，已知边际平均成本为$h(x)=-\frac{38.4}{x^2}+0.4$(万元/台)，其平均成本为11.64万元，求总成本C与产量x的函数关系.

解：边际平均成本为$h(x)=-\frac{38.4}{x^2}+0.4$，则平均成本为

$$\frac{C(x)}{x}=\int h(x)dx=\int(-\frac{38.4}{x^2}+0.4)dx=\frac{38.4}{x}+0.4x+C_0.$$

又

$$\frac{C(10)}{10}=3.84+4+C_0=11.64\Rightarrow C_0=3.8.$$

$\therefore$ 平均成本为

$$\frac{C(x)}{x}=\frac{38.4}{x}+0.4x+3.8,$$

$\therefore$ 总成本为

$$C(x)=0.4x^2+3.8x+38.4.$$

9. 某厂生产某种产品的边际利润函数为$L'(x)=x-3x^2$，其中x是产量，若生产两单位时，总利润为60，求总利润函数$L(x)$.

解：边际利润函数为$L'(x)=x-3x^2$，则总利润函数为

$$L(x)=\int(x-3x^2)dx=\frac{x^2}{2}-x^3+C_0,$$

而$L(2)=60\Rightarrow C_0=66$，故总利润函数为

$$L(x)=\frac{x^2}{2}-x^3+66.$$

10. 某厂生产某产品的边际费用为$k'(x)=x^2-10x+120$(元/件)，其中x为产量，已知生产6件时的总费用725元，求总费用函数$k(x)$.

解：总费用函数$k(x)=\int k'(x)dx=\int(x^2-10x+120)dx=\frac{x^3}{3}-5x^2+120x+C_0$，

$$k(6)=725\Rightarrow C_0=113,\ \therefore\ k(x)=\frac{x^3}{3}-5x^2+120x+113.$$